Math 129—Managerial Precalculus
Measurable Outcomes

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Reference text: Numbers in brackets refer to sections of Tan, *Applied Mathematics for the Managerial, Life, and Social Sciences,* seventh edition.

Note: Outcomes marked **(Optional)** may appear on the final exam with the unanimous consent of all instructors.

1. Graphing and lines

1(a) Define and determine slope of a line segment connecting two points in the plane. [2.1]

1(b) Identify a pair of line segments as parallel or perpendicular. [2.1]

1(c) Determine whether three points are collinear. [2.1]

1(d) Given some equation for a line, re-write the equation of the line in various forms: point-slope form, slope-intercept form, and general form. [2.2]

1(e) Find the equation of a line in various forms given any of the following information about the line: (i) two points, (ii) one point and the slope, or (iii) one point and the equation of another line, known to be either parallel or perpendicular to the line of interest. [2.2]

1(f) Recognize and write equations of horizontal and vertical lines. [2.2]

1(g) Recognize that the slope of a horizontal line is zero, while the slope of a vertical line is undefined. [2.2]

2. Functions of a single variable

2(a) Define the words *function, domain,* and *range.* [2.3]

2(b) Evaluate functions given in formula form when variables are replaced by numbers. [2.3]

2(c) Evaluate functions given in formula form when variables are replaced by literal expressions. [2.3]

2(d) Graph a function given a set of points. [2.3]

2(e) Use the vertical line test to decide whether a graph is the graph of a function. [2.3]

2(f) Determine domain and range of a function [2.3]

2(g) Recognize common forms which generate domain restrictions: zero denominator, even-index radicals. [2.3]

2(h) Define and graph piecewise-defined functions. [2.3]

3. Algebra of functions

3(a) Given a pair of functions, form the sum, difference, product and quotient functions [2.4]

3(b) Determine the domains of the above combinations. [2.4]

3(c) Form the composition of two functions. [2.4]

3(d) Recognize that composition is not multiplication and is not commutative. [2.4]

3(e) Write a given function as the composition of two simpler functions. [2.4]

4. Linear Functions and Economic Applications

4(a) Economic application: write the cost function, revenue function, and profit function, given sufficient economic information. [2.5]

4(b) Locate the intersection of two lines, given their equations. [2.5]

4(c) Determine the break-even point of linear cost and revenue functions. [2.5]

4(d) Find the production level corresponding to a specified profit or loss. [2.5]

5. Quadratic Functions and Quadratic Equations

5(a) Recognize a quadratic function in general form and its properties: domain, range, concavity, vertex, axis of symmetry and intercepts. [2.6]

5(b) **(Optional)** Convert a quadratic expression in general form to standard form, by completing the square. [2.6]

5(c) Locate the vertex of a parabola. [2.6]

5(d) Identify the axis of symmetry as an equation (not a number). [2.6]

5(e) Find the intercepts of a parabola. [2.6]

5(f) Distinguish between a quadratic expression and a quadratic equation. [2.6]

5(g) Use the discriminant to determine the number of x-intercepts and real roots. [2.6]

5(h) Apply the theory of quadratic functions to determine the maximum and minimum values of an economic or physical quantity. [2.6]

5(i) Model demand and supply curves as quadratics, and recognize that market equilibrium occurs at their point of intersection in the first quadrant. [2.6]

5(j) Locate the point of market equilibrium when supply and demand curves are quadratic, or one is linear and the other quadratic. [2.6]

5(k) Recognize polynomial functions, rational functions, and power functions. [2.7]

6. Exponential functions

6(a) Graph exponential functions with bases greater than 1 and also with positive bases less than 1. [3.1]

6(b) Identify the characteristics of the graph of an exponential function with base greater than one: continuity, smoothness, increasing behavior, concavity, and intercepts. [3.1]

6(c) Recognize the domain and range of exponential functions. [3.1]

6(d) Recognize the distinction between increasing and decreasing exponentials and the relevance to their applications. [3.1, 3.3]

6(e) Recognize that laws of exponents learned for integers can be extended to real numbers. [3.1]

6(f) Solve exponential equations when bases are equal, or can be made equal by a simple manipulation. [3.1]

6(g) **(Optional)** Solve exponential equations which are reducible to quadratic. [3.1]

7. The logarithm function

7(a) Define a logarithmic function as the inverse of an exponential function. [3.2]

7(b) Recognize the relationship between the graph of a logarithmic function and the graph of an exponential function with the same base. [3.2]

7(c) Recognize that logarithms are defined only for positive arguments, since exponentials are never negative or zero. [3.2]

7(d) Convert statements involving logarithms into equivalent statements involving exponentials, and vice-versa. [3.2]

7(e) Correctly utilize notation for common and natural logarithms. [3.2]

7(f) Graph logarithmic functions with various bases. [3.2]

7(g) For a logarithmic function, identify the domain, range, intervals of continuity, intervals of increase/decrease, and intervals of concavity. [3.3]

7(h) State the laws of logarithms. [3.2]

7(i) Expand the logarithm of a complex expression in terms of sums, differences and multiples of logarithms of simpler expressions by applying the laws of logarithms. [3.2]

7(j) Condense an expression containing sums and multiples of logarithms into the logarithm of a single quantity by applying laws of logarithms. [3.2]

7(k) Recognize that there is no rule to expand the logarithm of a sum or difference, or to condense a product or quotient of logarithms. [3.2]

7(l) Solve logarithmic equations, with awareness of the possibility of extraneous roots and hence the need for checking solutions. [3.2]

7(m) Solve exponential equations by taking logarithms, possibly with numerical approximations. [3.2]

7(n) Solve logarithmic equations by exponentiating, possibly with numerical approximations. [3.2]

8. Application of Exponential and Logarithmic Functions

8(a) Identify and construct exponential models, understanding the roles of Q_0 (the initial quantity) and k (the growth/decay constant). [3.3]

8(b) In exponential growth/decay problems, identify whether the time is given and the quantity is to be found or whether a quantity is given and the time is to be found, and solve the equation for the appropriate variable. [3.3]

8(c) Solve word problems involving exponential growth or decay, e.g. radioactive decay. [3.3]

8(d) Recognize that logistic growth function involves several independent variables, and solve for any one variable in terms of the remaining variables. [3.3]

8(e) Apply the logistic growth model to the spread of an infectious disease in a closed population; find the number infected given the time or the time required for a certain number to be infected. [3.3]

9. Interest Calculations in Finance (Single Deposit)

9(a) Calculate simple interest and accumulated amount; and recognize that accumulated amount is linear in time. [4.1]

9(b) For interest and accumulated interest calculations, solve for any one variable when given values for the rest. [4.1]

9(c) Calculate compound interest for accumulated amount; and recognie that accumulated amount is exponential in time. [4.1]

9(d) For compound interest calculations, solve for any one variable when given values for the rest. [4.1]

9(e) Recognize continuous compounding as an idealization of infinitely frequent conversion periods. [4.1]

9(f) For continuously compounded interest calculations, solve for any one variable when given values for the rest. [4.1]

9(g) Calculate the *effective rate of interest*, which is the rate of interest that, when compounded annually, will yield the same amount as the given rate compounded periodically. [4.1]

9(h) In compound interest problems, calculate the present value of a future amount. [4.1]

9(i) In continuously compounded interest problems, calculate the present value of a future amount. [4.1]

10. Annuities (Periodic Deposits/Withdrawals)

10(a) Distinguish single-investment problems from periodic investment problems. [4.2]

10(b) Calculate present value and future value of annuities. [4.2]

10(c) Calculate the periodic payment needed to attain a specified future value for (i) savings goal (ii) municipal bond sinking fund (ii) future capital expenditure. [4.3]

10(d) Recognize that home mortgage principal is the present value of the whole stream of monthly payments to be made. [4.2]

10(e) Apply amortizations to mortgages or other loans, which may also have a down payment of a fixed amount, or as a percentage of the total. [4.3]

10(f) **(Optional)** Estimate accumulated total and periodic payment by ignoring interest rate and multiplying or dividing. [4.2, 4.3]

10(g) **(Optional)** Show that for a given rate and principal, while the required periodic payment decreases with a longer term, it does not approach zero, as the limiting value is the interest-only case. [4.2, 4.3]

11. Systems of Linear Equations

11(a) Solve a system of two linear equations in two variables for cases where there is (i) a unique solution, (ii) no solution and (iii) an infinite number of solutions. [5.1]

11(b) Illustrate the three cases mentioned above by sketching graphs. [5.1]

11(c) Write the system of linear equations equivalent to a given word problem. [5.1]

11(d) Use the Gauss-Jordan elimination method with augmented matrices to solve a system of linear equations with a unique solution, by performing allowed row operations. [5.2]

11(e) Use the Gauss-Jordan elimination method with augmented matrices to identify when a system of linear equations has no solutions. [5.2]

11(f) Use the Gauss-Jordan elimination method with augmented matrices to identify when a system of linear equations has infinitely many solutions, and to write the solutions in parameterized form. [5.2]

12. Matrices, Matrix Operations, and Applications

12(a) Add and subtract matrices. [5.4]

12(b) Calculate the transpose of a matrix. [5.4]

12(c) Multiply matrices by scalar quantities. [5.4]

12(d) Evaluate expressions combining addition and subtraction with scalar multiplication. [5.4]

12(e) Solve matrix equations for unknown matrix or for letter variables in matrix elements. [5.4]

12(f) Multiply matrices. [5.5].

12(g) Demonstrate by example that matrix multiplication is not commutative. [5.5]

12(h) Write the identity matrix of size n. [5.5]

12(i) Write a system of linear equations as an equivalent matrix equation [5.5]

12(j) State and apply the formula for the inverse of a 2×2 matrix. [5.6]

12(k) Solve a system of linear equations by using inverse of a matrix. [5.6]

13. Systems of Linear Inequalities

13(a) Graph a system of any number of linear inequalities in two variables. Shade the solution set. Use solid or dashed lines for included or excluded points, respectively. Use a test point to locate proper half-plane for inclusion. [6.1]

13(b) Determine whether the solution set of a system of linear inequalities is bounded or unbounded. [6.1]

13(c) Write the system of linear equalities corresponding to given word problem. [6.2]

13(d) Use a graphical method for solution of linear programming problems by the method of corners. Find absolute maximum and minimum value of given objective function. [6.3]

13(e) Recognize conditions under which an absolute maximum or absolute minimum may exist. [6.3]